Ques:- What do you mean by the term degrees of freedom? State and prove the law of equipartition of energy.

Ans:-

Degree of freedom:-

The number of independent parameters or quantities require to specify the configuration of a system is known as its degree of freedom.

For example the motion of an idea simple pendulum as one degree of freedom.

An ant moving on the table two degree of freedom, since two co-ordinates (x, y) are needed to locate its position at any instant.

A beam has three degrees of freedom. Since three quantities (x, y, z) are required to specify its during its position.

This idea may be extended to the case of gas molecules a mono-atomic gas (He, A) molecules having translations motion only got three degrees of freedom.

In a diatomic gas (H_2, O_2 etc) molecule there is binding between the two atoms, hence there are three degree of freedom of translation and two degree of freedom of rotating. Thus it has all together five degrees of freedom.

In a triatomic gas (CO_2) molecules can rotate about any of the three co-ordinate axes. Hence it has six degrees of freedom (i.e. 3 translational and 3 rotational).

Law of Equipartition of Energy :-

Statement :-

The law states that the energy per degrees of freedom of a molecule is constant and equal to $\frac{1}{2}kT$.

Where k = Boltzmann's Constant
T = absolute temp of the system.

To establish this law we start with the expression for pressure exerted by a perfect gas.

This expression is

$$p = \frac{1}{3} m n c^2$$

where m = mass of molecule
n = number of molecule
c = r.m.s. velocity the molecule.

Now from above expression

we have $pv = \frac{1}{3}(nv)\, mc^2$ —— (1)

or, $pv = \frac{1}{3} N mc^2 = RT$ Gas law.

or, $\frac{1}{3} N m_1 c_1^2 = \frac{1}{3} N m_2 c_2^2 = RT$

where we consider two gas molecules of mass m_1 & m_2 and velocities c_1 & c_2. This relation is true according to Avogadro's hypothesis.

$\frac{1}{3} N mc^2 = RT$
$mc^2 = 3\frac{R}{N}T$
$= 3KT$
$\therefore K = \frac{R}{N}$

Therefore generally we can write

$$\frac{1}{2} mc^2 = \frac{3}{2}\frac{R}{N}T = \frac{3}{2} KT \quad \text{—— (3)}$$

The velocity, $c^2 = u^2 + v^2 + w^2$

$$\therefore \frac{1}{2} m(u^2 + v^2 + w^2) = \frac{3}{2} kT$$

At const. temp. the components of velocity are equal (since gas exert equal pressure in all direction).

$$\therefore \frac{1}{2} m(u^2 + v^2 + w^2) = \frac{1}{2} m 3u^2$$
$$= \frac{1}{2} m\, 3v^2$$
$$= \frac{1}{2} m\, 3w^2$$
$$= \frac{3}{2} kT$$

i.e. $\frac{1}{2} mu^2 = \frac{1}{2} mv^2 = \frac{1}{2} mw^2 = \frac{1}{2} kT$

Thus the energy for each degrees

of freedom per molecule is $\frac{1}{2}kT$.
This Statement is true for
kinetic energy of translation only.
If in addition there is vibra-
tional motion (say in the case of solid)
this also gives $\frac{1}{2}kT$ and hence
the total energy becomes kT.